

|  |  |  |
| --- | --- | --- |
| Mathematics Department | |  |
| Course: A1MAA | |
| Topic Title: Investigation 3 – Effects of scaling | |
| Student Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | Date: \_\_\_\_\_\_\_\_\_\_\_\_ | | |
| Special Instructions: Calculator Allowed | Time Allowed: 60 minutes | | |
|  | Marks: / 50 | | |

**EFFECTS OF SCALING ON VOLUME : IN-CLASS INVESTIGATION**

In this task the effects of changing dimensions on the volumes of 3-dimensional shapes are examined.

Formulae which may be useful: cube , sphere , cylinder 

**Question 1 (12 marks)**

(a) Complete the tables below. (7)

Volumes of cubes

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Length of side (cm) | Volume (cm3) | Side doubled (cm) | New volume (cm3) | Scale factor |
| 1 | 1 | 2 | 8 |  |
| 2 | 8 | 4 |  |  |
| 3 | 27 | 6 | 216 | 8 |
| 4 | 64 | 8 |  |  |
| 5 | 125 | 10 |  |  |

Volumes of spheres

|  |  |  |  |
| --- | --- | --- | --- |
| Radius (cm) | Volume (cm3) | Radius doubled (cm) | New volume  (cm3) |
| 1 | = 4.2 | 2 |  |
| 2 | = 33.5 | 4 | = 268.1 |
| 3 | = 113.1 | 6 |  |
| 4 | = 268.1 | 8 | = 2144.7 |
| 5 |  | 10 | = 4188.8 |

(b) How many times as large (the scale factor) is the volume of a cube after its sides are doubled? Justify your answer. (2)

(c) If a cube has a volume of 3375 cm3 and its sides are then doubled, what is the new volume? (1)

(d) What is the effect on the volume of a sphere when the radius is doubled? (1)

(e) If a spherical balloon has a volume of 14 137 cm3 and is deflated so that its diameter is halved, what will be the new volume? (1)

**Question 2 (13 marks)**

(a) Complete the table, rounding values to the nearest whole number. (5)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Volume of a cylinder** | | | | | |
| Radius  (cm) | Height  (cm) | Volume (cm3) | Radius doubled (cm) | New volume (cm3) | Scale  factor |
| 1 | 10 | 31 | 2 |  |  |
| 2 | 10 | 126 | 4 | 503 |  |
| 3 | 10 | 283 | 6 | 1131 |  |
| 4 | 10 | 503 | 8 | 2011 |  |
| 5 | 10 |  | 10 | 3142 |  |

(b) How many times as large is the volume of the cylinder when the radius is doubled and the height is kept at 10 cm? Give your answer to the nearest whole number and show how you determined the value. (2)

(c) Consider the situation where both the height and radius are doubled as shown in the table below. Values have been rounded to the nearest whole number.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Volume of a cylinder | | | | | |
| Radius  (cm) | Height  (cm) | Volume (cm3) | Radius doubled (cm) | Height  doubled  (cm) | New volume (cm3) |
| 3 | 10 | 283 | 6 | 20 | 2262 |
| 3 | 20 | 565 | 6 | 40 | 4523 |
| 3 | 30 | 848 | 6 | 60 | 6786 |
| 5 | 10 | 785 | 10 | 20 | 6283 |
| 5 | 20 | 1571 | 10 | 40 | 12 566 |
| 5 | 30 | 2356 | 10 | 60 | 18 850 |
| 7 | 10 | 1539 | 14 | 20 | 12 315 |
| 7 | 20 | 3079 | 14 | 40 | 24 630 |
| 7 | 30 | 4618 | 14 | 60 | 36 945 |

What happens to the volume of a cylinder when both the radius and the height are doubled? Give reasonable evidence to support your conclusion. (3)

(d) A cylindrical water tank has a volume of 14 m3. (3)

What would be the volume of a tank that was

(i) The same height but double the width?

(ii) Double the width and double the height?

**Question 3 (8 marks**)

The tables provided show the volumes of cubes for different side lengths.

|  |  |  |  |
| --- | --- | --- | --- |
| Side (cm) | Volume (cm3) | Side trebled (cm)  (multiplied by 3) | New volume (cm3) |
| 1 | 1 | 3 | 27 |
| 2 | 8 | 6 | 216 |
| 3 | 27 | 9 | 729 |
| 5 | 125 | 15 | 3375 |

|  |  |  |  |
| --- | --- | --- | --- |
| Side (cm) | Volume (cm3) | Side quadrupled (cm)  (multiplied by 4) | New volume (cm3) |
| 1 | 1 | 4 | 64 |
| 2 | 8 | 8 | 512 |
| 3 | 27 | 12 | 1728 |
| 5 | 125 | 20 | 8000 |

|  |  |  |  |
| --- | --- | --- | --- |
| Side (cm) | Volume (cm3) | Side x 5 (cm) | New volume (cm3) |
| 1 | 1 | 5 | 125 |
| 2 | 8 | 10 | 1000 |
| 3 | 27 | 15 | 3375 |
| 5 | 125 | 25 | 15 625 |

(a) By what factor is the volume changed when the sides are multiplied by

(6)

(i) 3 (ii) 4 (iii) 5

(iv) 10 (v) 0.5 (vi) *k*

(b) A cubic crate has a volume of 4.096 m3. What is the volume of another cubic crate with sides a quarter of the length of the sides of the first crate?

(2)

**Question 4 (8 marks)**

The tables provided show the volumes of spheres for different radii. Values have been rounded to the nearest tenth.

|  |  |  |  |
| --- | --- | --- | --- |
| Radius (cm) | Volume (cm3) | Radius trebled (cm) | New volume (cm3) |
| 1 | 4.2 | 3 | 113.1 |
| 2 | 33.5 | 6 | 904.8 |
| 3 | 113.1 | 9 | 3430.6 |
| 5 | 523.6 | 15 | 14 137.2 |

|  |  |  |  |
| --- | --- | --- | --- |
| Radius (cm) | Volume (cm3) | Radius quadrupled (cm) | New volume (cm3) |
| 1 | 4.2 | 4 | 268.1 |
| 2 | 33.5 | 8 | 2144.7 |
| 3 | 113.1 | 12 | 7238.2 |
| 5 | 523.6 | 20 | 33 510.3 |

|  |  |  |  |
| --- | --- | --- | --- |
| Radius (cm) | Volume (cm3) | Radius x 5 (cm) | New volume (cm3) |
| 1 | 4.2 | 5 | 523.6 |
| 2 | 33.5 | 10 | 4188.8 |
| 3 | 113.1 | 15 | 14 137.2 |
| 5 | 523.6 | 25 | 65 449.8 |

(a) By what factor is the volume changed when the radius is multiplied by

(6)

(i) 3 (ii) 4 (iii) 5

(iv) 10 (v) 0.1 (vi) *k*

(b) A spherical gas balloon has a volume of 195.43 m3. What will be the volume of the balloon if it deflates so that the radius is a third of the original length? (2)

**Question 5 (9 marks)**

(a) Use algebra and the volume formulae (cube , sphere  and cylinder ) to show (7)

(i) the change to the volume of a cube when the side is multiplied by *k.*

(ii) the change to the volume of a sphere when the radius is multiplied by *k.*

(iii) the change to the volume of a cylinder when the radius is multiplied by *k* and the height is multiplied by *t.*

(b) What do your answers to Part (a) of Question 5 suggest about your answers to the previous questions? (2)

**End of questions**